Search and Destroy: 
Competition between Search Engines 
and Consumer Prices

“The explosive growth of the Internet promises a new age of perfectly competitive markets. With perfect information about prices and products at their fingertips, consumers can quickly and easily find the best deals. In this brave new world, retailers’ profits margins will be competed away, as they are all forced to price at cost.”


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Abstract: This paper investigates the impact on prices of competition between Price-Comparison search engines. The presence of rivals may lead search engines to increase the number of vendors they list. However, it is unclear that consumers will benefit from this. There is a conflict of interests between types of consumers.

Key Words: Search Engines, Consumer Search, Price Levels
JEL Classification: D43, D83, L13
1 Introduction
1.1 Preliminary Thoughts

In this sub-section I layout the motivation of this research.

From the consumers’ perspective, one of the more promising aspects of e-commerce was that it would reduce search costs. With search engines, consumers could easily observe and compare the prices of a large number of vendors, and identify bargains. The consumers’ enhanced ability of comparing prices would discipline vendors, and put downward pressure on prices. According to an estimate, finding a high interest rate CD requires 25 minutes using the telephone, 10 minutes using the Internet, and less than a minute using a price-comparison search engine (Butler et al. (1997)). Brown & Goolsbee (2002) found that the increase in Internet use reduced the price of term life insurance by 8–15%, but only after price-comparison search engines were introduced, and for insurance types covered by the search engines.

Presumably, the larger the number of vendors whose price a search engine lists, and that thereby consumers can easily compare, the more competitive the market becomes. However, there are several technical reasons for search engines to: cover only a small subset of the Internet, and collect and report information biased in favor of certain vendors. Several studies confirm this perspective (Bradlow & Schmittlein (1999), Lawrence & Giles (1999, 1998)). Furthermore, this technology-induced tendency is reinforced by economic motives. Search engines are self-interested, profit seeking institutions, which draw most of their revenues from vendors, either in the form of fees for inclusion in the listings, commissions on referred sales, or advertising. The following case is revealing. On 2001, the Consumer Federation of America made a study of 25 Internet sites offering comparative term life insurance information. Only 28% of the sites displayed the true lowest-price insurer; 20% of the sites displayed a lowest-price insurer which was 4.4% more expensive; 16% of the sites had even higher lowest quotes, up to 33% more expensive; and 36% of the sites were inadequate. Although these sites contend that they provide the lowest-cost insurance,

1 See Appendix A for a brief description of search engines, and some commercial arrangements between search engines and vendors.
2 The search literature has no simple prediction about the relation between search costs, price levels, or price dispersion (Pereira (2002a) and Samuelson & Zhang (1992)).
3 Most book price-comparison search engines sample between 20 and 40 vendors, whereas a query under booksellers at Yahoo shopping returns a list of 266 vendors, which probably is only a subset of all vendors online. On 2001, Commercial Alert filed a complaint with the Federal Trade Commission, against the search engines, AltaVista, AOL, Direct Hit, iWon, LookSmart, Microsoft, and Terra Lycos, and requested an investigation on whether these vendors were violating federal prohibitions against deceptive acts and practices, by inserting advertisements in search engine results without clear and conspicuous disclosure that the advertisements are advertisements. The concealment or obfuscatory disclosure of paid placement and inclusion could mislead search engine users to believe that search results are based on relevancy alone, not on marketing ploys, and could ultimately affect consumers’ purchasing decisions, by diverting their attention to the advertisers.
some charge commissions on insurance sales, and therefore do not include no commission insurers, which are usually low-cost providers.

In this paper I investigate the impact on consumer prices of competition between *Price-Comparison search engines*. I address two questions. *First*, does the presence of more rivals induce search engines to increase the number of vendors they list at their site? And *second*, does an increase in the number of vendors search engines list put downward pressure on prices? The answer to these questions is mixed. Competition may lead search engines to increase the number of vendors they list. However, it is unclear that consumers will benefit from this.

1.2 The Building Blocks


*First*, price-comparison search engines draw their revenues only from vendors. This assumption reflects the reality of the industry. There are many possible business models for the price-comparison search engine industry. One example are price clubs, which charge consumers rather than vendors. But all business models that emerged so far rely on vendors to generate revenues.

*Second*, products are branded, and consumers differ in their preferences for the various brands. This assumption reflects the findings of several empirical studies. Smith & Brynjolfsson (2001) documented the importance of brand, for homogeneous products, and among users of price-comparison search engines, who are presumably the most price sensitive consumers online. Using data for searches for books conducted at Dealtime.com in late 1999, they report that while price is the strongest predictor of customer choice, only 49% of customers choose the cheapest vendor. Among consumers who do not choose the cheapest offer, the average selected offer was 20% higher than the cheapest offer. Consumers were willing to pay 5% more to purchase from Amazon, rather than from the lowest priced vendor, and 3% more to purchase from Barnes & Noble or Borders. Johnson et al. (2001) using data from

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*Ellison & Ellison (2001) raise concerns of anti-competitive behavior at price-comparison sites. Their focus is on the behavior of vendors.*
Media Metrix from July 1997 through June 1998, report that 70% of CD shoppers, 70% of the book shoppers, and 42% of the travel shoppers, were observed as being loyal to just one site through the duration of the his data. On average, households that browse a category initially visited only 1.1 book sites, 1.2 CD sites, and 1.8 travel sites.5

Third, vendors can post different prices at their homepages, and at a search engine. With the exception of Baye & Morgan (2002) and Nahm (2002), the prevailing assumption in the literature has been that vendors cannot price discriminate. However, if products are branded, and if consumers differ in their preferences for the various brands, then from the vendors’ perspective, having a price posted at a search engine serves the dual purpose of: (i) competing for consumers without a preferred brand, and (ii) price discriminating.6

Fourth, vendors choose prices cognizant of which rivals they face at a search engine. Baye & Morgan (2001) assume that vendors simultaneously choose their prices, and whether to post them at the search engine. This implies that vendors choose prices without knowing the identity of their rivals at the search engine. While this might have been true at the beginning of the industry, it is implausible in the longer run. Besides, it is likely that a vendor’s decision to post a price at a search engine reflects a long run strategic position, which changes less frequently than the price itself.

1.3 Overview of the Paper

In this sub-section I give an overview of the model, and the paper’s main results.

Consider a market for a functionally identical, branded search good, where there are: (i) a finite number of search engines, (ii) a large but finite number of vendors, and (iii) a large number of consumers.

Search engines charge a Submission fee to include a vendor in their database, i.e., to Index a vendor. An indexed vendor will have its price listed by the search engine in response to a query for the product. Search engines set submission fees to maximize profit.

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5 For other studies that found evidence of the importance of brand online see (…)

6 Discriminating between sales generated by the homepage and the search engine is a mild form of price discrimination. Internet technology and customer databases make it easy to estimate demand elasticity and identify customers, which create limitless opportunities for price discrimination. The consumers’ perception of unfair treatment may constrain the vendors’ behavior (Sinha (2000)). Amazon tested dynamic pricing over the 2000 Labor Day weekend (Mendelson & Meza (2001)). Users in a chat room on the DVDtalk.com Web site noticed that some Amazon customers paid more than others for the same DVDs. One person reported that he ordered a DVD for $24.49, and the following week he saw that the price increased to $26.24. When he deleted Amazon’s “cookies” from his computer, the price fell to $22.74. The discovery was quickly publicized over the Internet, leading to numerous customer complaints. Amazon denied that it engaged in dynamic pricing, claimed it was only testing consumer responses to different discount levels, and issued a prompt apology and refunds for 6,000 customers.
Vendors make three decisions. First, they choose whether to be indexed by a search engine. Second, they choose a price to post at their homepage. Third, they choose a price to post at a search engine, if they choose to be indexed by one. The prices posted at the homepage and the search engine may differ.

For consumers, acceding to a search engine’s site involves no monetary cost, but involves an utility loss, an Access cost, which differs across consumers. There are three types of consumers. Switchers have: a reservation price of one, an access cost of zero, and view all brands as perfect substitutes. Weak Loyal have: a reservation price of one, a heterogeneous access cost, and a preferred brand. Strong Loyal have: a reservation price higher than one, a prohibitive access cost, and a preferred brand. Consumers do not know the prices of individual vendors, and can only learn them by either: (i) visiting the vendors’ homepages, or (ii) visiting the site of a search engine that indexes the vendors.

In equilibrium, switchers accede to the search engine where they expect to find the lowest price, and buy at the lowest posted price. Strong loyals buy at the homepage of their preferred brand. Weak loyals with a high access cost, buy at the homepage of their preferred brand; weak loyals with a low access cost, buy through the search engine that indexes their preferred brand. Within a search engine, vendors randomize between charging a higher price and selling only to their weak loyals, and charging a lower price to try to sell also to switchers. At their homepage, vendors either charge the reservation price of weak loyals, or the reservation price of strong loyals, depending on the relative proportions of strong and weak loyals. A vendor’s incremental profit for being indexed by a search engine decreases with the number of vendors present at the search engine. Equating the submission fee to the vendors’ incremental profit for being indexed, gives the vendors’ inverse demand for a search engine’s services.

A monopolist search engine reduces quantity, i.e., the number of indexed vendors, to increase the submission fee. An increase in the number of search engines present in the industry, i.e., an increase in Competition, leads search engines to reduce the submission fee and index a larger number of vendors.

Competition among search engines impacts the vendors’ prices posted at search engines through three effects. First, given the proportion of switchers and weak loyals that visit a search engine, an increase in the number of indexed vendors, i.e., in the number of rivals with which a vendor has to compete to sell to switchers, reduces the probability that an indexed vendor will have the lowest price at the search engine. This reduces the marginal benefit
of charging a price lower than the weak loyals’ reservation price. Vendors respond by shifting probability mass from lower to higher prices. The price distribution at a search engine shifts to the right. Second, given the proportion of switchers that visit a search engine, the shift to the right of the price distribution at a search engine, increases the expected price paid by weak loyals at a search engine. This causes the proportion of weak loyals that choose to visit a search engine to decrease. Vendors respond by reducing the lower bound of the support of the price distribution at the search engine, and by shifting probability mass from higher to lower prices. Third, as the number of search engines increases, switchers distribute themselves through a larger number of search engines, and the proportion of switchers per search engine decreases. Vendors respond by increasing the lower bound of the support of the price distribution at the search engine, and by shifting probability mass from lower to higher prices. The net impact of the first two effects increases the expected price paid by weak loyals that visit a search engine, and decreases the expected price paid by switchers. For weak loyals, the third effect reinforces the other two, but for switchers the third effect pushes in the opposite direction of the other two. Thus, an increase in the number of search engines increases the expected price paid by weak loyals present at a search engine, and may decrease or increase the expected price paid by switchers. Taking only consumers into account, one can see that the welfare effects of competition among search engines are potentially ambiguous.

The results of this paper rely on the way price distributions at search engines vary with the number of vendors. In particular, they depend on an increase in the number of indexed vendors reducing: (i) the probability of an indexed vendor having the lowest price, (ii) the proportion of weak loyals that visit a search engine, and (iii) the proportion of switchers that visit a search engine. Several empirical and experimental studies are in line with these predictions. Iyer & Pazgal (2001) using data collected at 5 price-comparison search engines (Mysimon, BottomDollar, EvenBetter, Bsilly and PriceScan) report evidence that within a search engines: (i) vendors take turns at charging the lowest price, which suggests that vendors mix on prices, (ii) the expected price increases with the number of vendors, and (iii) the expected minimum price decreases with the number of vendors. Arbatskaya & Baye (2001), using data collected over a 3 month period from Microsurf.com, a price comparison search engine for mortgage rates, report that: (i) over 90% of the vendors offered the lowest mortgage rate at least once in their local market, and (ii) 7% of
the vendors changed their rates over 50% of the time. This high turnover in the identity of the lowest priced vendor again suggests that vendors mix on prices. Morgan et al (2001) and. Cason & Friedman (2000) (...).

1.4 Literature

In Section 2 I present the model, and in Section 3 I characterize it’s equilibrium. In Section 4 I conduct the analysis of the model for the cases in which there is one, and several search engines. Section 5 considers several generalizations, and Section 6 concludes. Appendix A contains a description of search engines and commercial arrangements between search engines and vendors, and Appendix B contains the proofs of the paper’s results.

2 The Model
In this section I present the model.

(a) The Setting
Consider an electronic market for a functionally identical, branded search good, which opens for 1 period. There are: (i) \( m \geq 1 \) price-comparison search engines, (ii) \( n \geq 3 \) vendors, and (iii) many consumers. Subscript \( j = 1, \ldots, m \) refers to search engines, and subscript \( i = 1, \ldots, n \) refers to vendors.

[Insert Figure 1 here]

The game has 5 stages (Figure 1). In stage 1 price-comparison search engines set submission fees. In stage 2 vendors decide whether to be indexed by a price-comparison search engine. In stage 3 vendors choose which price to post at their homepage. In stage 4 vendors choose which price to post at the search engine’s site, if they chose to be indexed by one. In stage 5 consumers search and buy. Then delivery takes place, agents receive their payoffs, and the market closes.
(b) Search Engines

A *Price-Comparison search engine* in response to a search for the product, lists the URLs found in its index, and the prices charged. Search engine $j$ charges a *Submission Fee*, $z_j$ on $\mathbb{R}^+_0$, to index a vendor. Denote by $k_j$, the number of vendors that search engine $j$ indexes. I will refer to $k_j$ as the *Size of the Index* of search engine $j$. Search engines give consumers free access to their sites. Denote by $\kappa$ on $(0,1)$, the constant marginal cost of indexing a vendor, which is identical across search engines. Denote by $\Lambda_j = (z_j - \kappa)k_j$, search engine $j$’s profit. In Section 5 I discuss how fixed costs could be introduced in the model. To simplify exposition, I assume that at least 1 search engine is active.

A search engine’s *strategy* is a submission fee. A search engine's *payoff* is its profit.

(c) Vendors

Vendors are identical and risk neutral. Their marginal costs are constant and equal to zero.

For tractability I assume vendors can only join 1 search engine. All players observe the vendors’ decisions of weather to be indexed (SEE).

I assume that $n \geq \sum_{j=1}^m jk_j$, which ensures that vendors will not be rationed by search engines (SEE).

Denote by $q_i$, the price vendor $i$ posts at its homepage; and denote by $p_i$, the price vendor $i$ posts at search engine it was indexed by, if it chose to be indexed. Prices $q_i$ and $p_i$ may differ. In general, the price equilibrium in stage 4 will involve mixed strategies. For expositional simplicity I assume that vendors choose prices $q_i$ on $[0,1+v]$ and choose prices $p_i$ on $[0,1]$ (SEE).

Denote by $\Pi_i^h(q_i)$, the expected profit of vendor $i$ for the sales generated by its homepage; denote by $\Pi_j^h(p_i)$, the expected profit of vendor $i$ for the sales of generated by search engine $j$, by which it was indexed; and denote by $\Pi_i(q_i, p_i) = \Pi_i^h(q_i) + \Pi_j^h(p_i)$, the total expected profit of vendor $i$.

A vendor’s *stage 2 strategy* is a decision of whether to be indexed by a search engine, given submission fees; a vendor’s *stage 3 strategy* is a price to post at its homepage, given (…); and a vendor’s *stage 4 strategy*, if it
choose to be indexed, is a cumulative distribution function over the prices to post at the search engine’s site, \( F_{ij}(.) \), given (...). Denote by \( \underline{p}_{ij} \) and \( \bar{p}_{ij} \), the lowest and highest prices on its support of \( F_{ij}(.) \), respectively. A vendor’s 

**payoff** is its total net expected profit.

**(d) Consumers**

There is a unit measure continuum of risk neutral consumers. Each consumer has a unit demand.

Some consumers have a preferred brand and other consumers do not. Consumers with a preferred brand view the other brands as an unacceptable substitute to their preferred. In an alternative interpretation, these consumers are locked-in with a prohibitive switching cost. Consumers without a preferred brand view all brands as perfect substitutes.

For consumers, acceding to a search engine’s site might involve an utility loss, an **Access cost**, denoted by \( \alpha \), because they might have to download software, learn to use the search engine’s interface, configure the interface, wait for the data to be downloaded, etc.\(^8\) The access cost differs across consumers, due to differences in computer skills, or the opportunity cost of time. Acceding to a vendor’s homepage is costless.

There are 3 types of consumers, which differ with respect to: (i) their preferred brand, (ii) their valuation of the good, and (iii) the cost of acceding to a search engine. **Strong Loyals** have: a preferred brand, a reservation of \( 1 + v \), where \( v \) belongs to \((0, +\infty)\), and a prohibitive access cost. Each vendor has a proportion of strong loyals of \( \lambda \) on \((0,1)\). **Weak Loyals** have: a preferred brand, a reservation of \( 1 \), and their access cost is uniformly distributed on \([0,1]\). Each vendor has a proportion of weak loyals of \( \omega \) on \((0,1)\). **Switchers** have: no preferred brand, a reservation of \( 1 \), and an access cost of \( 0 \). The proportion of switchers is \( \sigma \). To sum up, the consumers’ preferences are given by the indirect utility function:

\[
V = \begin{cases} 
  r - q_i - \delta & \iff \text{buy from vendor } i \text{ at home page} \\
  r - p_{ij} - \alpha - \delta & \iff \text{buy from vendor } i \text{ at site of search engine } j \\
  0 & \iff \text{don’t buy}
\end{cases}
\]

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\(^7\) The **Universal Resource Locator**, URL, is the address of an Internet resource.

\(^8\) For Dealtime.com the modal waiting time between a query being placed and prices being displayed is 45 seconds, and 10% of queries timeout at 3 minutes, whereas the model response time for individual vendors is of 2 seconds (Montgomery et al (2001)).
where: (i) $r = 1$ for switchers and weak loyals, and $r = 1 + v$ for strong loyals; (ii) $\alpha = 0$ for switchers, $\alpha = +\infty$ for strong loyals, and $\alpha$ is uniformly distributed on $[0,1]$, for weak loyals; (iii) $\delta = 0$ for switchers or for consumers whose preferred brand is $i$, and $\delta = +\infty$ for consumers whose preferred brand is not $i$.

Consumers do not know the prices of individual vendors, and can only learn them by either: (i) visiting a vendor homepage, or (ii) visiting a search engine’s site. For expositional simplicity I assume it is impractical for consumers to visit both a vendor homepage and a search engine’s site. Consumers hold common beliefs about the cumulative distribution of the price posted by vendor $i$ at search engine $j$, denoted by $\Phi_j(.)$.

For Weak Loyals and Switchers, a stage 5 strategy is a rule, $s$, which says (...). A consumer's payoff is its expected surplus, net of the access cost.

(e) Equilibrium
An equilibrium is: (...).

3 Characterization of Equilibrium
In this section I construct the equilibrium by working backwards. Switchers buy at a search engine. Strong loyals buy at the homepage of their preferred brand. Some weak loyals buy at the homepage of their preferred brand, and other weak loyals buy at the search engine that indexes their preferred brand. At a search engine, vendors randomize over prices. At their homepage, vendors sometimes charge the reservation price of weak loyals, and other times charge the reservation price of strong loyals. Vendors choose to be indexed by the search engine that offers the highest net incremental profit. A monopolist search engines charges the submission fee for which marginal revenue equals marginal cost, and oligopolist search engines set a submission fee equal to marginal cost.

3.1 Stage 5: Consumers Decide Whether to Visit a Search Engine and Buy
In this sub-section I characterize the consumers equilibrium behavior.

Denote by $\varepsilon_{ij} := \int p \, d\Phi_j$, the price weak loyals expect to pay vendor $i$ at search engine $j$.

9 Allowing consumers to do both would not change the equilibrium, but would complicate the exposition.
Given that their access cost is prohibitive, \( \alpha = +\infty \), strong loyals visit the homepage of their preferred brand. If offered a price no higher than \( l + \alpha \), they accept the offer and buy; otherwise they reject the offer and exit the market.

The next Lemma characterizes the equilibrium behavior of switchers and weak loyals.

**Lemma 1:** (i) If their preferred brand \( i \) was indexed by a search engine \( j \), then weak loyals visit the site of the search engine \( j \) if \( \varepsilon_{ij} + \alpha \leq q_i \); otherwise they visit the homepage of vendor \( i \). If offered a price no higher than \( l \), either at the site of the search engine \( j \), or at the homepage of vendor \( i \), they accept the offer and buy; otherwise they reject the offer and exit the market. (ii) Switchers visit the site of the search engine where they expect to pay the lowest price. At the site of a search engine, they buy at the lowest posted price, if it is no higher than \( l \); otherwise they reject the offer and exit the market. In case of a tie switchers choose at random.

Weak loyals are indifferent between visiting the homepage of their preferred brand and visiting the search engine that indexes their preferred brand, if visiting the homepage generates the same expected utility as visiting the search engine: \( l - q_i = l - \varepsilon_{ij} - \alpha \). Denote by \( \alpha_{ij} := q_i - \varepsilon_{ij} \), the access cost for which a weak loyal is indifferent between visiting the site of search engine \( j \), which indexed its preferred brand \( i \), and visiting the homepage of vendor \( i \). The measure of weak loyals that visit search engine \( j \) is \( \alpha_{ij}\alpha \). The smaller the price weak loyals expect to pay at the search engine, the larger the proportion of them that chooses to visit the search engine. The measure of weak loyals that visit the homepage of vendor \( i \) is \( (l - \alpha_{ij})\alpha \).

### 3.2 Stage 4: Vendors Set Prices at the Search Engines

In this sub-section I characterize the vendors’ equilibrium pricing behavior at search engines.

Denote by \( S_j \), the proportion of switchers that visit search engine \( j \). Denote by \( \hat{p}_{ij} \), the minimum price charged by any vendor other than vendor \( i \) at search engine \( j \), and denote by \( \hat{m}_{ij} \), the number of vendors charging
The profit function of vendor $i$ for the sales generated by search engine $j$, when it charges price $p_i$ is:

$$\pi_i(p_i) = \begin{cases} 
    p_i \left( \alpha_i \omega + S_j \right) & \iff p_i < \min \{ \hat{p}_i, 1 \} \\
    p_i \left( \alpha_i \omega + S_j \hat{p}_i \right) & \iff p_i = \hat{p}_i \leq 1 \\
    p_i \alpha_i \omega & \iff \hat{p}_i < p_i \leq 1 \\
    0 & \iff 1 < p_i 
\end{cases}$$

Ignoring ties, the expected profit of a vendor that charges $p \leq 1$ at search engine $j$ is:

$$\Pi_i(p) = p\alpha_i \omega + pS_j \Pr[p < \hat{p}_i] \quad (1)$$

Denote by $b_j$, the lowest price a vendor is willing to charge at search engine to sell to its weak loyals, and to the switchers that visit the search engine, i.e., $b_j \left[\alpha_j \omega + S_j \right] - \alpha_j \omega = 0$.

Next I will distinguish between 2 cases: (i) $\alpha_{ij} = \alpha_j$, for all $i$ indexed by $j$, and (ii) $\alpha_{ij} \leq \alpha_{2j} \leq \ldots \leq \alpha_{kj}$.

The next 2 **Lemmas** state some auxiliary results. For case $\alpha_{ij} = \alpha_j$ I will drop subscript $i$, whenever it is unnecessary.

**Lemma 2:** Let $\alpha_{ij} = \alpha_j$, for all $i$ indexed by $j$. Then, for all $i$ indexed by $j$: (i) $\bar{p}_{ij} = \bar{l}$. (ii) $p_{ij} = b_j = b_j$. (iii) $\Pi_{ij} = \alpha_j \omega$. (iv) $F_j$ has a connected support. (v) $F_j$ is continuous on $[b_j, 1]$.

If $\alpha_{ij} = \alpha_j$, at a symmetric equilibrium, i.e., $F_j(.) = F_j(.)$, for all $i$ indexed by $j$, $\Pr[p < \hat{p}_{ij}] = \left[ 1 - F_j(p) \right]^{i-1}$. Thus, using **Lemma 3:** (iii), in equilibrium:

$$p\alpha_j \omega + pS_j \left[ 1 - F_j(p) \right]^{i-1} = \alpha_j \omega \quad (2)$$

The next **Lemma** characterizes the equilibrium price distribution at search engine $j$, for the case where $\alpha_{ij} = \alpha_j$, for all $i$ indexed by $j$. 


Lemma 3: Let $\alpha_{ij} = \alpha_j$, for all $i$ indexed by $j$. Then:

$$F_j(p, \omega, S_j, k_j, \alpha_j) = \begin{cases} 
0 & \iff p < b_j \\
1 - \left(\frac{\alpha_j \omega}{S_j} \left(\frac{1-p}{p}\right)^{k_j-1}\right) & \iff b_j \leq p < 1 \\
1 & \iff 1 \leq p
\end{cases}$$

$b_j(\omega, S_j, \alpha_j) = \frac{\alpha_j \omega}{\alpha_j \omega + S_j}$

In the symmetric case, the price equilibrium in Stage 4 is similar to that of Varian (1980) (see also Burdett & Judd (1983), Rosenthal (1980), and Stahl (1989)).

Lemma 4: Let $\alpha_{ij} \leq \alpha_j \leq \ldots \leq \alpha_{k,j}$, for all $i$ indexed by $j$. See Baye, Kovenock & deVries, (1992)

3.3 Stage 3: Vendors Set Prices at Their Homepages

In this sub-section I characterize the equilibrium prices that vendors charge at their homepages.

The profit function of vendor $i$ for the sales of generated by its homepage, when it charges price $q_i$ is:

$$\pi^h_i(q_i) = \begin{cases} 
\lambda + (1 - \alpha_j) \omega & \iff q_i \leq 1 \\
(1 + v) \lambda & \iff 1 < q_i \leq 1 + v \\
0 & \iff q_i > 1 + v
\end{cases}$$

The next Lemma characterizes the equilibrium prices that vendors charge at their homepages

Lemma 5: At its homepage, if $\omega (1 - \alpha_j) \geq v \lambda$, then a vendor charges $q^*_i = q^*(\omega, \lambda, v, \alpha_j) = 1$; otherwise a vendor charges $q^*_i = q^*(\omega, \lambda, v, \alpha_j) = 1 + v$.

The expression for $\pi^h_i(q_i)$ and Lemma 5 apply whether a vendor is indexed or not, noting that in the latter case $\alpha_j = 0$.

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10 I show constructively that equilibrium exists. Alternatively, existence follows from Theorem 5 of Dasgupta & Maskin (1986). The equilibrium described in the proposition is the unique symmetric equilibrium. There is also a continuum of asymmetric equilibria (Baye, Kovenock, & deVries (1992)).
Weak loyals will only buy at the homepage of their preferred brand if \( q^* = 1 \); if \( q^* = 1 + v \) they prefer not to buy; in either case \( V = 0 \). Thus, \( \alpha_j = l - \varepsilon_j \), and therefore \( F_j(p; \omega, S_j, k_j, \varepsilon_j) = F_j(p; \omega, S_j, k_j, \alpha_j) \) and \( b_j(\omega, S_j, \varepsilon_j) = b_j(\omega, S_j, \alpha_j) \).

[Insert Figure 2 here]

An increase in the proportion of weak loyals that visit the search engine that indexed its preferred brand, \( a_j \), caused possibly by a decrease in the expected price, \( \varepsilon_j \), may cause a vendor to switch from charging \( q^* = 1 \) to charging \( q^* = 1 + v \). Thus, 3 cases may occur. \textbf{First}, vendor \( i \) may charge \( q^* = 1 + v \), both if it is indexed or not. Denote by \( \Gamma^h := \{(\omega, \lambda, v, \varepsilon_j): \omega < v\lambda\} \), the set of parameter values for which this case occurs (Figure 2). \textbf{Second}, vendor \( i \) may charge \( q^* = 1 \) if it is not indexed, and charge \( q^* = 1 + v \) if it is indexed. Denote by \( \Gamma' := \{(\omega, \lambda, v, \varepsilon_j): (l - \alpha_j)\omega \leq v\lambda < \omega\} \), the set of parameter values for which this case occurs. \textbf{Third}, vendor \( i \) may charge \( q^* = 1 \), both if it is indexed or not. For this case, which occurs if \( v\lambda < (l - \alpha_j)\omega \), the search industry is not viable, in the sense that no vendor would pay a positive submission fee to be indexed. So from now on I will focus on \( (l - \alpha_j)\omega \leq v\lambda \).

\[ \text{3.4 Stage 2: Vendors Decide Whether to be Indexed by a Search Engine} \]

In this sub-section I characterize the vendors’ equilibrium behavior with respect to becoming indexed.

Denote by \( \Delta \Pi_{ij} \), the \textbf{Incremental Profit} of vendor \( i \) for being indexed by search engine \( j \), i.e., the difference between vendor \( i \)’s profits when it is indexed by search engine \( j \), and when it is not. For the case \( \Gamma^h \), the incremental profit is: \( \alpha_j \omega + (l + v)^{-} - (1 + v)\lambda = \alpha_j \omega \). Similarly for case \( \Gamma' \). Thus, the incremental profit of vendor \( i \) for being indexed by search engine \( j \) net of the submission fee, \( z_j \), is:

\[
\Delta \Pi_{ij}(z_j; \omega, \lambda, v, \varepsilon_j) = \begin{cases} 
\alpha_j \omega - z_j & \text{on } \Gamma^h \\
\alpha_j \omega + (v\lambda - \omega)^{-} - z_j & \text{on } \Gamma'
\end{cases}
\]

The next \textbf{Lemma} characterizes the vendors’ equilibrium behavior with respect to becoming indexed.
Lemma 6: A vendor should choose to be indexed by the search engine that offers the largest non-negative net incremental profit for being indexed; otherwise a vendor should choose not to be indexed.

The net incremental profit of being indexed is larger for case $\Gamma^h$ than for case $\Gamma^l$, since in the latter case $\lambda < 0$.

In equilibrium the consumers’ beliefs about the distribution of the price posted by vendor $i$ at search engine $j$ are correct, $G(p) = F_j(p; \omega, S_j, k_j, \epsilon_j)$. Thus, $F_j(p; \omega, S_j, k_j, \epsilon_j)$, $b_j(\omega, S_j, k_j, \epsilon_j)$, and $\Delta \Pi_g(z_j; \omega, \lambda, v, S_j, k_j)$.

A vendor’s incremental profit, i.e., its marginal evaluation of being indexed by a search engine, depends on how many other vendors the search engine indexes, $k_j$. And as we will see ahead, a vendor’s marginal evaluation of being indexed decreases with the size of the index.

3.5 Stage 5: Search Engines Set Submission Fees
In this sub-section I characterize the equilibrium submission fees. I analyze first the case where there is a single search engine, and then the case where there are several search engines.

3.5.1 Stage 5: The Monopoly Case
In this sub-section I characterize the equilibrium submission for the case of a monopolist search engine.

Let $m = 1$. I will omit superscript $j$ for the monopoly case. Obviously $S = \sigma$. Let $\theta := (\omega, \lambda, \sigma, v)$. Equating the net incremental profit, $\Delta \Pi_i(z; \theta, k)$, to zero, defines implicitly function $z = Z(k; \theta)$, which gives the maximum submission fee a vendor is prepared to pay to be indexed by the search engine, given that the search engine indexes $k$ other vendors. I interpret $Z(.)$ as the search engine’s Inverse Demand function. Similarly, I will interpret $k = K(z; \theta) := \left( Z(z, \theta) \right)^{-1}$ as the search engine’s Demand function.

Using $k = K(z; \theta)$, the monopolist search engine’s problem becomes:

$$\max_z \left( z - \kappa \right) K(z; \theta)$$

The next Lemma characterizes the equilibrium submission fee of a monopolist search engine.
Lemma 7: In equilibrium, a monopolist search engine sets a submission fee for which the marginal revenue equals marginal cost.

Denote by $z^m(\theta, \kappa) := \arg\max_z \left( z - \kappa \right) K(z; \theta)$, the Monopoly Submission fee, i.e., the submission fee that maximizes the profit of a monopolist search engine. And denote by $k^m(\theta, \kappa) := K(z^m; \theta)$, the Monopoly index size.

Similarly, $F^m(p; \omega, \sigma, k^m)$, $b_j(\omega, \sigma, k^m)$, and $e^m(\omega, \sigma, k^m)$, represent the price distribution at the search engine, the lower bound of the support of the price distribution at the search engine, and the expected price paid by weak loyals at the search engine, respectively, for the case of the monopoly search engine.

3.5.2 Stage 5: The Oligopoly Case

In this sub-section I characterize the equilibrium submission for the case where there are several search engines.

Let $m > 1$. As in the case of the monopolist search engine, equating the net incremental profit, $\Delta \Pi_j(z_j; \theta, k_j)$, to zero, defines implicitly function $z_j = Z(k_j; \theta)$, which I interpret as search engine $j$’s Inverse Demand function.

And similarly, I interpret $k_j = K_j(z_j; \theta) := (Z_j)^{-1}(z_j; \theta)$ as search engine $j$’s Demand function.

The next Lemma characterizes the equilibrium submission fees for oligopolist search engines.

Lemma 8: If $z_j = z_{j'} = z^o$, for all $j$, $j'$, then $z^o = \kappa$.

Corollary 1: If $z_j = z_{j'} = z^o$, for all $j$, $j'$, then for all $j$: (i) $k_j = k^o$, (ii) $S_j = \sigma/m$, and (iii) $e_j = e^o$.
4 Analysis

In this section I conduct the analysis of the model.

4.1 The Driving Forces

In this sub-section I discuss the economic effects that drive the results of this paper.

Rewrite (2) as:

$$\frac{pS_j}{\left[1 - F_j(p)\right]^{k_j - 1}} = \frac{\alpha_j}{\omega_j}$$

(3)

If vendor $i$, indexed by search engine $j$, charges a price $p$ lower than the reservation price of weak loyals, it has the lowest price at search engine $j$ with probability $\left(1 - F_j\right)^{k_j - 1}$, it sells to $S_j$ switchers, and earns an additional expected profit of $pS_j\left(1 - F_j\right)^{k_j - 1}$: the Volume of Sales effect. However, it loses $\left(1 - p\right)$ per weak loyal, and a total of $\left(1 - p\right)\alpha_j$: the per Consumer Profit effect. The Volume of Sales effect is the marginal benefit of charging a price lower than the weak loyals’ reservation price, and the per Consumer Profit effect is the marginal cost. These 2 effects drive this paper’s results.

4.2 Some Preliminary Results

In this sub-section I present some auxiliary results.

Denote by $\hat{F}_j := 1 - \left[1 - F_j\right]^{k_j}$, the distribution of the minimum price charged at search engine $j$, which indexes $k_j$ vendors. And denote by $\mu_j := \int p\hat{F}_j$, the expected price paid by switchers at search engine $j$. The expected price paid by switchers, $\mu_j = b_j + \int_{b_j}^1 \left(1 - F_j\right)^{k_j - 1} dp$, is lower than the expected price paid by weak loyals, $\varepsilon_j = b_j + \int_{b_j}^1 \left(1 - F_j\right) dp$.

The next Proposition analyzes the impact of changes in the size of the index, $k_j$, on: (i) the expected price paid by weak loyals at a search engine, $\varepsilon_j$, (ii) the expected price paid by switchers, $\mu_j$, (iii) the measure of weak loyals that visit the search engine, $\alpha_j\omega_j$, and (iv) a vendor’s net incremental profit for being indexed, $\Delta\Pi_j$. (see if there are asymmetric equilibria)
**Proposition 1:** An increase in the size of the index: (i) increases the expected price paid by weak loyals at a search engine, (ii) decreases the expected price paid by switchers, (iii) decreases the measure of weak loyals that visit the search engine, and (iv) decrease a vendor’s net incremental profit for being indexed.

[Insert Figure 3 here]

Inspection of (3), shows that an increase in the size of the index, $k_j$, has 2 impacts. **First**, given the measure of weak loyals that visit the search engine, an increase in the size of the index increases the number of rivals with which a vendor has to compete to sell to switchers from $k_j - 1$ to $k_j$. This decreases the probability that an indexed vendor will have the lowest price, $(1 - F_j)^{k_j-1}$, which decreases the Volume of Sales effect, i.e., the marginal benefit of charging a price lower than the weak loyals’ reservation price. This first direct impact leads vendors to shift probability mass from lower to higher prices, and causes the distribution to shift to the right, i.e., in the first-order stochastic dominating sense (Figure 3 (a)). **Second**, the first-order stochastic dominating shift in the price distribution, caused by the direct impact, raises the expected price paid by weak loyals at the search engine. This decreases the measure of weak loyals that visit the search engine, $\alpha_{j,0} = (t - \varepsilon_j)\alpha_j$, which reduces the per Consumer profit effect. This second indirect impact leads vendors to decrease the lower bound of the support, and to shift probability mass from higher to lower prices, which causes the distribution to kind of rotate (Figure 3 (a)). The total impact of the increase in the size of the index is to cause the price distribution to rotate clock-wise (Figure 3 (b)).

(SEE) The decrease in the lowest price vendors charge, $b_j$, reduces the expected price paid by switchers. However, the expected price by weak loyals that visit the search engine increases. Recall that vendors now charge higher prices with a larger probability. Switchers and weak loyals that visit a search engine have conflicting interests with respect to the size of the index. Switchers prefer a large index and weak loyals that visit a search engine prefer a small index.

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11 See Guimarães (1996) for a related discussion.
12 See Morgan, Orzen & Sefton (2001) for a related discussion.
A vendor’s net incremental profit for being indexed, $\Delta \Pi_j$, decreases with the size of the index, $k_j$, because the measure of weak loyals that visit the search engine, $\alpha_j \omega = (1 - \varepsilon_j) \omega$, decreases with the size of the index. It means that for a vendor becoming indexed is less valuable, the larger the number of other vendors also indexed by the same search engine. This suggests the existence of a **Network Diseconomy** within search engines.

The next **Lemma** analyzes the impact of the measure of switchers present at a search engine, $S_j$, on the expected price paid by switchers, $\mu_j$, and the expected price paid by weak loyals that visit the search engine, $\varepsilon_j$.

**Lemma 9:** A decrease in the measure of switchers present at a search engine increases both the expected price paid by switchers and the expected price paid by weak loyals that visit the search engine.

A decrease in the measure of switchers present at a search engine, $S_j$, reduces the **Volume of Sales** effect, which leads vendors to shift probability mass from lower to higher prices, in the first-order stochastic dominating sense. As a consequence, both the expected price paid by switchers, $\mu_j$, and the expected price paid by weak loyals that visit the search engine, $\varepsilon_j$, increase (Stahl (1989)).

### 4.3 Competition Between Search Engines and Consumer Prices

In this sub-section I analyze the impact of the presence of rival search engines on the size of indices, and on consumer prices.

The next **Proposition** orders the monopoly index size, $k^m$, and the oligopoly index size, $k^o$, and orders the expected price paid by weak loyals at a search engine when the search engine industry is a monopoly, $\varepsilon^m(\omega, \sigma, k^m)$, and when the search engine industry is an oligopoly, $\varepsilon^o(\omega, \sigma|m, k^o)$.

**Proposition 2:** (i) The oligopoly index size is larger than the monopoly index size. (ii) The expected price paid by weak loyals that visit a search engine is larger if the search engine industry is an oligopoly than if the search engine is a monopoly.
A monopolist search engine reduces the size of its index to increase the submission fee, whereas oligopolist search engines price at marginal cost. As a consequence the size of the index of each search engine present in the industry is smaller under monopoly than under oligopoly, $k^m < k^o$. A monopolist reduces quantity to increase price.

The expected price paid by weak loyals at a search engine when the search engine industry is a monopoly, $\varepsilon^m(\omega, \sigma, k^m)$, and the expected price paid by weak loyals at a search engine when the search engine industry is an oligopoly, $\varepsilon^o(\omega, \sigma/m, k^o)$, differ in the underlying size of the index of the search engine, $k_j$, and the measure of switchers present at the search engine, $S_j$. For the expected price paid by weak loyals, from Proposition 1: (i) and Lemma 9, both effects push in the same direction. As a consequence, the expected price paid weak loyals that visit the search engine is higher under oligopoly than under monopoly, $\varepsilon^m(\omega, \sigma, k^m) < \varepsilon^o(\omega, \sigma/m, k^o)$.

The ordering of the expected price paid by switchers when the search engine industry is a monopoly $\mu^m(\omega, \sigma, k^m)$, and the expected price paid by switchers when the search engine industry is an oligopoly, $\mu^o(\omega, \sigma/m, k^o)$, is potentially ambiguous, because from Proposition 1: (ii) and Lemma 9, the size of the index and the measure of switchers present at the search engine push in opposite directions.

The next Proposition analyzes the impact on the number of search engines present in the industry, $m$, on the expected price paid by weak loyals at a search engine, when the industry is an oligopoly, $\varepsilon^o(\omega, \sigma/m, k^o)$, and on the expected price paid by switchers, when the industry is an oligopoly, $\mu^o(\omega, \sigma/m, k^o)$.

**Proposition 3: (i)** The expected price paid by weak loyals that visit a search engine, if the search engine industry is an oligopoly, increases with the number of search engines present in the industry. (ii) The expected price paid by switchers, if the search engine industry is an oligopoly, increases with the number of search engines present in the industry.

(...)

(effects of shifts in $\kappa$, and $\theta$)
5 Extensions

5.1 Fixed Costs

5.2 Asymmetric Vendors

5.3

6 Conclusion

Appendix A: Search Engines

In this appendix I make a brief description of search engines, and some commercial arrangements between search engines and vendors. This account is intended to be informative rather than rigorous, and is based on numerous sources, among which (…).

A Search Engine is a program that accesses and reads Internet pages, stores the results, and returns lists of pages, which match keywords in a query. It consists of three parts: (i) a crawler, (ii) an index, and (iii) the relevancy algorithm. The Crawler, or spider, is a program that automatically accesses Internet pages, reads them, stores the data, and then follows links to other pages. The Index, or catalog, is a database that contains the information the crawler finds. The Relevancy Algorithm is a program that looks in the index for matches to keywords, and ranks them by decreasing order of relevance. Relevance is determined through criteria such as: link analysis, or, click-through measurements. This description refers to crawler-based systems, such as Google or AltaVista. There are also Directories, such as Yahoo, in which listings are compiled manually. Yahoo also uses as a back up the services of Inktomi. Price-Comparison search engines, also known as shopping agents or shopping robots, are a class of search engines, which crawl commercial Internet sites. In addition to addresses from vendors, they also collect and display other information like prices, or return policies. There are also price-comparison directories.

There are several types of commercial arrangements between search engines and firms. Under Paid Submission, search engines crawl an advertiser’s site sooner than normal, with no guarantee that they will index the site, or if they do, of where the site's pages will be placed in the search results. Under Paid Placement, search engines place a link to an advertiser’s site near the top of the search results. Under Paid Inclusion, search engines deep list an advertiser’s site, i.e., crawl beyond the homepage of an Internet site, and list other pages in additional categories. It does not guarantee a position in the search results, but it interacts directly with the editorial results, and makes an advertiser more likely to appear in response to a wide range of queries. Under keyword-linked Banner Advertisements, and Content Deals, search engines promote an advertiser's content on their search results, usually in a separate area from the editorial results. Yahoo displays a Barnes & Noble window whenever a book related query is placed. Some search engines also license their software. The revenue of a typical price-comparison search engine is 40% from fees for placements, 50% from banner advertisements, and 10% from referred sales (Bob Tedeschi, “E-commerce Report: Comparison Sites Struggling as Well”, New York Times, February 5, 2002).

Appendix B
In this appendix I prove the main text’s results.

References


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The first case occurs when $\omega$ is small and $\lambda$ is large. The second case occurs when $\omega$ and $\lambda$ take intermediate values. And the third case occurs when $\omega$ is large and $\lambda$ is large.
The first impact causes the distribution to shift from A to B, and the second impact causes the distribution to shift from B to C.

An increase in the size of the index leads the price distribution at the search engine to rotate clock-wise.
The search engine is an intermediary between rms and consumers: 1See Evans (2008) for an interesting presentation of the online advertising industry, with a special emphasis on search engines.Â In section 6, I build upon my baseline model to incorporate the issue of competition between search engines, and that model exhibits a new effect, namely that the presence of several search engines can lead to a rise in the advertising price, which is passed through to consumers, if rms are not able to price discriminate consumers based on the search engine they.Â 1. Search engine pricing: The search engine chooses a per-click fee $a$, which is publicly observed by rms and consumers. 2. Firms pricing and targeting: Firms decide whether to register on the search engine. The many different search engines on the web are optimized to different things in different context. Pick the engine that meets your needs, not your habits.Â Google is the reigning leader of spartan searching and is the most used search engine in the world. Google is fast, relevant, and the most extensive single catalog of web pages available. Try Google images, maps, and news features; they are outstanding services for locating photos, geographic directions, and headlines. A search engine is a software system that is designed to carry out web searches (Internet searches), which means to search the World Wide Web in a systematic way for particular information specified in a textual web search query. The search results are generally presented in a line of results, often referred to as search engine results pages (SERPs). The information may be a mix of links to web pages, images, videos, infographics, articles, research papers, and other types of files. Some search engines Meta search engines are search engines using other search engines. Meta searchers usually don't maintain their own crawling infrastructure. They do not search the web themselves, but contact other search engines for search result pages and assemble completely different (hopefully better) search results. A Web search engine is a web-based tool that enables users to locate information on the World Wide Web. Popular examples of search engines are Google, Yahoo!, and MSN Search. The information gathered by the spiders is used to create a searchable index of the Web. There are many browse