In this theoretical research paper we describe a collaborative effort between researchers and university faculty to improve how teachers are prepared to teach middle school mathematics. Two powerful instructional frameworks, UDL and TPACK are dovetailed within in a web-based dynamic textbook, “Proportional Dynabook” that focuses on proportional reasoning concepts related to ratio, similarity, and linear function. Theoretical tensions between special and general education teacher preparation programs influence the ongoing design of Proportional Dynabook. Pre-service and in-service special education teachers used Proportional Dynabook in a graduate level methods class to design a ratio lesson for a student who struggled with the concept. Teachers developed deeper understanding of ratio and related pedagogical strategies that make the content of mathematics accessible to diverse learners.

Classrooms are more diverse than ever due to recent legislation such as No Child Left Behind and the Individuals with Disabilities Education Improvement Act that require all students to have access to the general education curriculum and to be included in the same assessments (Van Garderen, Scheuermann, Jackson, & Hampton, 2009). To meet the increasingly diverse needs of students with learning differences who are being educated in the general education classroom, special education services are being brought to the general education classroom. The lines between general and special education are blurred and teachers are being asked to collaborate to meet the educational needs of an extremely diverse group of learners. These collaborative efforts are based on the idea that each teacher has specific knowledge and expertise that address the instructional needs of the class (Van Garderen et al., 2009). However, collaboration in theory does not address the friction that exists in practice. Philosophical differences in pedagogy and learning theory create tension in the design of instruction. While this tension can be an obstacle to the design of effective instruction, it can also be the vehicle that brings together special and general education researchers to conceive and design innovative instructional tools and practices to meet the needs of all students.

Students with high achievement, with learning disabilities and English Language learners are all receiving mathematics instruction from teachers who have been trained as math teachers or special education teachers but not usually both. Special education teachers tend to focus on pedagogy and interventions while general education teachers focus more on the content and structure of mathematics (Van Garderen et al., 2009). Now both teachers are given the formidable task of “raising the floor by expanding achievement for all, and lifting the ceiling of
achievement to better prepare future leaders in mathematics...(Friesen, 2008, p. 51). The prevailing special education service delivery model has the special education teacher providing instructional supports within the general education classroom. Hence, the special education teacher remains focused on pedagogy and intervention and the general education teacher remains focused on the math content. Often the special education teacher is attending only to the needs of students with Individual Education Plans (IEPs) and who have formally qualified for special education services thereby underutilizing the additional teaching resource in the classroom. This model may prevail because the special education teacher shies away from the more challenging math content and there is little time in the day for both teachers to do collaborative planning. However, in light of diminishing resources and budget cuts, full utilization of teaching resources must become the norm if we are indeed to raise the floor and ceiling of mathematics academic achievement.

We can begin to make changes in the way we prepare both general and special education teachers to teach mathematics. In teacher preparation and professional development programs, researchers and teacher educators from the field of general and special education must model collaborative practices and create dynamic learning communities. Silverman and Clay (2010) suggest that teacher candidates need social experiences where they can engage with mathematics in a way that that encourages the development of “deep, connected, unpacked mathematical understandings.”(p. ) Further, the authors suggest that the collaborative use of technology serves to establish the social environment necessary to promote higher levels of thinking and the development of deeper broader understandings of mathematics and pedagogy. Likewise, Shreyar, Zolkower, & Perez (2010) suggest that the “text” created through dialogue, written response, and diagram, in a social environment such as the classroom, can transcend the actual text in use. In this way, the original text serves as a vehicle for collectively making meaning of complex mathematical concepts, problems, and related pedagogy. The instructor in these models works to orchestrate the individual students in the class in such a way that, despite a beginning heterogeneous perception of mathematical concepts and pedagogy, the class ends up with a collective understanding of problems and approaches to solve them.

In this theoretical research paper we describe our innovative approach to engaging pre-service teacher candidates in mathematical and pedagogical thinking. First, we describe our collaborative development of Proportionality Dynabook, a web-based dynamic textbook aimed at supporting both special and general education pre-service teachers in their efforts to teach middle school mathematics. Second, we describe the frameworks of Universal Design for Learning (UDL) and Technological, Pedagogical and Content Knowledge (TPACK), which serve as the structural environment of our dynamic text. Third, we explain the curricular activity system designed to create a learning environment where pre-service teachers utilize this dynabook in a way that encourages social and cognitive engagement in issues surrounding middle school mathematics instruction for diverse learners.

Dynabook.
We draw inspiration from Alan Kay’s 1972 concept for a tablet-shaped personal computer (see, e.g., Kay & Goldberg, 1977), which he called a Dynabook. Although futuristic visions for the impact of computing stretch back much farther than Kay and have been updated many times since, we see Kay’s concept as a distinctive turning point – and the essence of Kay’s Dynabook is at the heart of the possibilities now emerging for digital texts in mathematics. Kay’s vision, however, requires more than mobile devices and the Web 2.0 capabilities that enable social networking, because Kay also wanted people to more easily and frequently engage with powerful ideas. Like Seymour Papert (1993), Kay saw the dynamic, interactive capabilities of computational media as opening up new ways to playfully engage with powerful ideas – through activities like programming, interacting with visualizations, exploring mathematical models, and playing with simulations.

The present project is a collaboration between SRI International, San Diego State University, San Francisco State University, CAST, Inc., and Inverness. SRI International is a research firm with expertise in math education and innovative uses of technology. San Diego State University and San Francisco State University prepare teachers to serve California students in general and special education, respectively. CAST, Inc. is a leader in educational technology and Universal Design for Learning (UDL). Inverness is a leading research and evaluation firm. Collectively, we have a unique combination of expertise in mathematics, math education, special education, teacher training, technology supported math instruction, UDL pedagogy, and evaluation, which has been productive in designing and building the first iteration of the Proportionality Dynabook.

As a team of researchers and educators, we are exploring and developing Kay’s ideas through a mathematics dynabook intended for use by prospective middle school mathematics pre-service teachers and centered on a key middle school concept of proportionality. Just as the strategies of summarization and prediction (e.g., in comprehending reading literature), and questioning and modeling (e.g., in learning from a science text), are essential in comprehension, we believe that there are similar strategies essential in mathematics. These include exploring the same mathematics through multiple equivalent entry points and making connections among different representations. Through our collaborative research with mathematics teacher educators and special education experts, we determined that the goals for this book should be to (a) support and reward the candidates when engaging more deeply in mathematical thinking, (b) encourage candidates to draw connections among related concepts of proportionality, and (c) develop teacher candidates’ awareness of potential student misconceptions and instructional options they could choose to support student development of more robust ideas about proportionality.

To support these ends, and cue these strategies, the fundamental structure of the “book” is not a linear narrative but rather a 3x3 matrix that can be navigated through different pathways. The columns of the matrix are concepts in three related strands of middle school mathematics that develop students’ proportional thinking: ratio (in the number strand), similarity (in the geometry strand), and linear function (in the algebra strand). Although many middle school mathematics teachers do not recognize this, these three concepts are deeply connected. For
example, “slope” is the ratio of two sides (the height and the width) of a slope triangle, and the application of triangles to the slope relies on the concept of similarity. By presenting these three concepts side by side, the Proportionality Dynabook aims to encourage teachers to explore these sorts of connections.

The rows of the Proportionality Dynabook are organized by three different “entry points” for engaging teachers in the mathematics. Teachers can become familiar with the mathematics first by exploring “challenging problems” – mathematics problems that are designed to be challenging to candidates and help them develop their own mathematical thinking. Candidates can further explore the mathematics by watching “video cases” of student thinking as students solve problems with ratio, similarity and linear functions. Finally, candidates can also explore lessons that are specially designed to take advantage of the dynamic medium of the Proportionality Dynabook by presenting mathematical ideas in a visual and interactive format. For example, in the linearity section, candidates develop the idea of linear function as they explore the relationship between timing of thunder and lightning reaching a campsite. This rich matrix of related concepts and related ways to encounter the concepts takes unique advantage of the ability to construct a “book” that does not need to have a strictly linear ordering of pages. In addition, a concept of an expert “tour” is planned, which can overlay a step-by-step trajectory on the book when it is desirable to guide candidates through the book in a linear order.

With our Proportionality Dynabook we seek to further develop the concept of the “book” as a social medium that enhances interaction between an instructor and his or her pre-service teacher candidates. For example, instructors can create “assignments” in the Proportionality Dynabook for their pre-service teachers, support activities of both reading and writing, allow pre-service teachers to engage deeply with powerful ideas, and extend their benefits to a diversity of learners. By dovetailing the UDL and TPACK framework in the design of the Proportionality Dynabook, we seek to immerse pre-service teachers in use of this resource as an exemplary instance of technology-rich mathematics in three ways: to deepen their own content knowledge, to develop key pedagogical skills, and to support beneficial use of technology in teaching.

**Universal Design for Learning (UDL).**

Universal Design for Learning supports the needs of diverse learners by providing multiple means of representation, expression, and engagement (Rose & Meyer, 2006; Rose, Meyer, & Hitchcock, 2005). The core idea of UDL is to embed supports in the medium, which learners can optionally activate when they need support to continue their progress. The benefit of UDL as a framework is that it provides a research-based taxonomy of the kinds of supports that learners may need and that technology can provide. Learners need supports for (a) connecting multiple representation of important ideas, (b) interacting with ideas and expressing ideas in new ways, and (c) maintaining a high-level of engagement.

With regard to these three kinds of supports, UDL concisely summarizes a vast literature on the brain, learning, and the role of technology. For example, UDL suggests that providing for multiple representations of a concept not only enables deeper engagement with that concept but
also enables access for a broader range of learners (citation). In the Proportionality Dynabook, UDL is introduced through videos, text, and diagrams as a framework to interpret the digital text. These features are embedded to give teachers ideas about how to support student access to and engagement with challenging mathematics. For example, a glossary is available which defines unfamiliar words and definitions by using a mixture of pictures and words. Multiple means of expression are available to users who can highlight words or sections in the Proportionality Dynabook and take notes in the margins. Further, when answering a question, they can write, draw a picture, explain verbally (into a microphone), or upload a file. To enhance engagement, “Stop and Think” prompts are strategically embedded in the text to encourage candidates to process the text more deeply. These features of UDL are combined with digital resources to increase TPACK in pre-service teachers.

**Technological, Pedagogical, and Content Knowledge (TPACK)**

Emerging technological advances combined with Schulman’s (1987) work on pedagogical content knowledge have led to the technological pedagogical and content knowledge (TPACK) framework (Mishra & Koehler, 2006). To increase TPACK in teacher candidates, technology needs to be strategically introduced and utilized by the instructor. It is not sufficient for students to access the Proportionality Dynabook without guidance and thoughtful pedagogy. In addition to careful instruction, content must be rich and challenging for candidates to increase understanding of the complexities of TPACK. Dynabook contains many access points to the three components of the instructional environment: technology, pedagogy and content. It incorporates features of UDL through digital technology and multiple means of expression. Videos of student thinking are assigned to candidates so they have a framework for pedagogy. The various videos will guide the candidate’s developing pedagogy when designing instruction to address student misconceptions. The content aspect of TPACK is distributed through many parts of the Dynabook, especially through the challenging problems for candidates. These are guided problems that include prompts to help them access the underlying mathematics through “How Do I Say It?” and “Get Me Started” features when working through the problems. There are also several dynamic representations involved in showing the solution of the problems when students check their work. The next section focuses on how the Dynabook was presented to teacher candidates who are working towards a special education teacher credential.

**Dynabook in a Teacher Education Classroom.**

The first year of this project was dedicated to planning the design and development of a dynamic web-based text that could improve how pre-service teachers learn middle school mathematics content and related pedagogy. The diverse research backgrounds of team members contributed to lively discussions and ongoing iterations of our evolving Proportional Dynabook. In its current form, we have a completed section on ratio. In an effort to examine how teacher candidates can learn mathematics content, specifically ratio, and related pedagogy with the use of Proportional Dynabook, we introduced our tool in an advanced level methods class for special education teachers at an urban university in Northern California. In California, special education
teachers are awarded a credential to teach at the K-12 grade level after successfully completing a series of graduate level courses. Special education interns work as the teachers of record while they attend the university in the evening to complete credential coursework. Most courses center around assessment, pedagogy, legalities and literacy with only one course devoted to mathematics. Since teacher educators in classes such as advanced methods often teach specific strategies, they choose content areas that tend to be more accessible to new teachers, leaving mathematics methods to the mathematics professors in the general education department where special education teachers take their one math class. Mathematics is an area that is frequently overlooked because it is considered difficult and less interesting than other topic areas. It is our hope that the Proportional Dynabook will provide a fun and interesting way for teacher educators, who are not experts in proportionality, to engage their pre-service teachers more fully in the concepts and pedagogy related to teaching proportional reasoning to diverse learners.

Thirteen pre-service and intern special education teacher candidates participated in two classes dedicated to interacting with the ratio section of Proportional Dynabook. Participants had varying levels of mathematics proficiency and teaching experience. During the first session, we introduced Dynabook with a scavenger hunt activity through various sections of Ratio (i.e., Introduction, UDL, Challenging Problems, and Video Cases). In addition, our teacher candidates watched two instructional videos related to the shifts in proportional reasoning outlined by Lobato, Ellis, Charles and Zbiek (2010).

During the first observation of the Dynabook training and proportionality development session, all the researcher observers agreed that the teachers seemed a bit overwhelmed by all the new material and technology. In addition to learning a new technological tool, and a new mathematics framework (Labato et al., 2010), they were also asked to think about middle school ratio problems that they found difficult to solve. Many might call these problems simple, but the students still had difficulty thinking through these problems and coming up with answers and novel ways to approach the problems. During a debriefing after the first session, we realized that the teachers were uncertain about their role in relation to the Dynabook. Questions like, “Who is this for?,” and “Would I use this with my students?” made us realize that we did not provide enough background information about the development and purpose of Dynabook. In addition, we had not provided a rational for these teachers to truly engage with Dynabook in the way that we had hoped they would. It is our contention that if a special education teacher candidates does not have a reason to engage with challenging mathematics content, he or she will not and instead attend to the many other responsibilities required by a job or coursework. For the second session, we designed a specific curricular activity that would provide purpose and rationale for our teacher candidates to engage with Proportional Dynabook.

In the beginning of the second three hour class session, the teacher candidates finished trying to solve the ratio problems and post answers to the communal whiteboard in Dynabook. While one of the researchers used the large screen computer in the lab, each teacher candidate followed along in their own Dynabook on individual screens. The anonymous postings on the whiteboard initiated a lively discussion of ratio concepts and common misconceptions shared
by several of the teacher candidates in the class. We described the rationale and purpose of this session by setting up a scenario where they were going to have to teach ratio to middle school mathematics starting tomorrow. Specifically, they were going to prepare to address the learning needs of a student struggling with the concept of ratio as depicted in one of the videos in Dynabook. Each pair viewed the video of Kayla, and began to discuss her misunderstandings along with their own confusions and sometimes partial understanding of ratio. As we observed, pairs moved in and out of the different sections of Dynabook utilizing various features to improve their own understanding so that they could develop a script to address Kayla’s level of understanding. Each pair remained engaged in discussion until the end of class.

In the video, Kayla, a middle school student, attempts to solve a ratio problem by incorrectly utilizing a procedural algorithm. Despite her answer not making sense, Kayla defends her use of the algorithm, representing a fairly typical procedural approach to proportionality in the absence of understanding. After carefully watching the video, the teacher candidates worked in pairs to create a lesson that addresses Kayla’s conceptual misunderstandings. Each pair created a script for the lesson and use that script to create a movie of the lesson using Xtranormal, a free, web-based software for created animated movies. We explained that by creating a script and movie, each pair must think about what words and language they will use to explain ratio. Then, by watching the movies, we have a way to share and discuss changes in the candidates’ thinking about ratio and the pedagogy they used. As the groups were working on the movie, we told them they were free to take a break, but most groups did not stop. By the end of class, we observed that not only were the teacher candidates able to talk more precisely about ratio, they were also able to talk about how to assess student understanding of ratio. They discussed how to define what level of thinking the student is operating at and ways to get them to the next level. This is the type of engagement we had hoped to initiate with the introduction of Dynabook.

One of the researcher observers commented “that we have a way to really engage pre-service students in UDL and TPCK -- it was completely evident to me that the candidates were really in that space with their thinking throughout the evening. We need to remember to describe both the tool --- the Dynabook -- and the curricular activity system in which it exists. Both are necessary to achieve the reasoning we saw …. It is not our story that the Dynabook causes great pre-service classroom experiences’ -- instead that the Dynabook is an enabler for a curricular activity system that increases candidate's social and cognitive engagement in these important issues. The Curricular Activity System language may be unfamiliar. We should capture both the tool's contribution and the activity design contribution -- that's the heart of it.”

As we continue to develop and test the Proportional Dynabook, we hope to share our ideas and collect feedback from those interested in improving how we prepare teachers to think about and teach mathematics. Our next steps include formal testing and data collection to more thoroughly examine how the Proportional Dynabook can expand meaningful engagement and develop teachers’ and students’ sense of connections across the middle school mathematics curriculum.
References.
Learning mathematics with understanding is essential. Mathematics today requires not only computational skills but also the ability to think and reason mathematically in order to solve the new problems and learn the new ideas that students will face in the future. Learning is enhanced in classrooms where students are required to evaluate their own ideas and those of others, are encouraged to make mathematical conjectures and test them, and are helped to develop their reasoning skills. A key challenge that mathematics teachers face in enacting current reforms is to orchestrate whole-class discussions that use students' responses to instructional tasks in ways that advance the mathematical learning of the whole class (e.g., Ball, 1993; Lampert, 2001). In particular, teachers are often faced with a wide array of student responses to cognitively demanding tasks and must find ways to use them to guide the class toward deeper understandings of significant mathematics. Here, we propose a model for the effective use of student responses to such tasks in whole-class discussions that teacher preparation. Which learning experiences matter most in preparing teachers to teach challenging mathematics? What should the balance be across the three areas of mathematics, mathematics education, and general pedagogy? The mathematics knowledge of US future middle school mathematics teachers generally is very weak compared to future teachers in Taiwan and Korea. It is also weak compared to Section I Teaching Mathematics: Foundations and Perspectives chapter 1 Teaching Mathematics in the Era of the NCTM. J. V. D. Walle, K. Karp, Jennifer M. Bay-Williams. Published 2012.