PROBLEM SOLVING, TEACHING, AND MORE: TOWARD A THEORY OF GOAL-DIRECTED BEHAVIOR

Alan H. Schoenfeld
University of California, Berkeley

Abstract: My 1985 book Mathematical Problem Solving offered a framework for analyzing how and why people are successful (or not) when they engage in problem solving—but, it didn't offer a theory that explained how and why people made the choices they did. Such a theory is now within reach. Solving a mathematical problem, teaching a lesson (or a year's course), and building a theory of problem solving are all examples of goal-directed behavior. I'll try to make a case that such behavior can be explained on the basis of models of individuals' knowledge, goals, beliefs, and a particular form of decision-making. In addition, this account will be consistent with what is known about learning and development, context and identity, and more.

Problem Solving, Teaching, and More: Toward a Theory of Goal-Directed Behavior

As its title suggests, this plenary presentation will be focused largely on theory. My goal is to develop a theoretical perspective for describing human activity in complex activities including mathematical problem solving and teaching (and in many others as well). The idea is that, as in the relationship between medicine and public health, a theoretical understanding of “how things work” can help to improve practice, although (because of so many other factors!) it does not guarantee that there will be any improvements.

The evolution of medicinal practice is a guiding metaphor for the work. Over the course of the 20th century and into the 21st, medicine evolved from a “folk art” into a practice with a scientific basis. The mind is more complex than the body, so we should expect progress in mathematics education to take as long a time. This talk describes a “hundred year plan.” Both the cognitive sciences and mathematics education began to coalesce in the 1970s, and we have made spectacular progress in the 40 or so years since then. (But there is still much to do!) Here is a rough chronology of that progress:

1970’s: Emergence of mathematics education, cognitive science
1980’s: A framework describing reasons for success or failure in problem solving
1990’s: Models of tutoring, on the way to a theory and models of teaching, and
2000’s: A proposed synthesis: a theory of goal-oriented “acting in the moment,” including problem solving and teaching

I also wish to address, at least through references to upcoming publications, the larger themes announced in the CIEAEM conference discussion document. Theme 2, Problem solving and institutionalization of knowledge, asks, “What place is given to problem solving in each or your countries? How were the research findings integrated in the curriculum? What are the key questions that are now studied?” Readers interested in these questions should consult a forthcoming special issue of the Zentrollblatt fur Didaktik der Mathematik (January 2008), edited by Gunter Toerner, Alan Schoenfeld, and Kristina Reiss. The situation concerning research and practice in the United States is more fully described in Schoenfeld (2008).
Recall the state of the art in the 1970s: Mathematics education was in its infancy; research methods were almost exclusively statistical; Piaget and constructivism were largely ignored or misinterpreted; and cognitive science was not yet a field. It was all we could do to make sense of a 20-minute problem solving session in the laboratory.

By 1985 things had changed. In my 1985 book *Mathematical Problem Solving* (MPS), I argued that the following four categories are necessary and sufficient for understanding the quality (and success) of problem solving attempts:

(i) the knowledge base  
(ii) problem solving strategies (heuristics)  
(iii) “control:” monitoring and self-regulation, or metacognition  
(iv) beliefs, and the practices that give rise to them

(Details will be given in the talk.) Thus, by 1985, we knew what “counted,” in the sense that we could explain, post hoc, what accounted for success or failure. BUT, there was a lot this framework didn’t do. The goals were established (i.e., “solve this problem”); the tasks didn’t change while you worked on them; and social interactions and considerations were negligible. More importantly in theoretical terms, MPS offered a framework, not a theory. Above and beyond pointing out what’s important, a theory provides rigorous explanations of how and why things fit together.

What I want, and what I have been working on for the past 20 years, is a theoretical approach that explains:

• how and why people make the choices they do,  
• while working on problems they care about,  
• amidst dynamically changing social environments.

The ideal domain for the development of such a theoretical approach is… teaching! Specifically, the teacher enters classroom with certain knowledge and goals. Sometimes conducting a lesson is easy; one goes through what has been planned. But sometimes it’s not, and the teacher has to adapt on the spot (problem solving!) That is: teaching is a dynamic goal-oriented problem solving activity in a complex social context. It is knowledge-based and often routine, but sometimes calls for urgent on-the-spot decisions. Indeed, so do most people’s jobs! The goal is to come up with a theoretical architecture that explains people’s decision-making during all these activities.

I claim that goal-oriented “acting in the moment” – including activities such as problem solving, tutoring, teaching, cooking, and brain surgery – can be explained and modeled by a theoretical architecture in which the following are represented:

• Knowledge  
• Goals  
• Orientations (an abstraction of beliefs, including values, preferences, etc.)  
• Decision-Making (according to an “internal calculus” that can be modeled as a form of cost-benefit analysis).

In broadest terms, the theory brings together the following main strands of research from the past 40 years. There has been research on knowledge and knowledge organization, especially in psychology and in mathematics education; on goal-oriented behavior, especially in fields such as artificial intelligence; on the role of beliefs in shaping human behavior; and in decision-making (as in studies of subjective decision-making, as in economics). (And, of course, there is the research on teaching and problem solving; see, e.g., Schoenfeld, 1985, 1998, 2002, 2006.)

Let me, briefly, indicate the importance of the categories bulleted above; then I will indicate how they fit together. In my presentation I work through some examples in fine detail (although the
real detail requires a book – each of two papers that model a teacher’s decision-making is about 100 pages long!).

By way of preface, here are some “top level” comments. Much of what we do in our daily lives (in almost all professions) is routine, and depends on access to routine knowledge. This routine is periodically punctuated by crucial decision-making – something takes place that requires an important choice, and that choice, by establishing what one intends to do (one’s goals), determines the course of events for some time to come. Moreover, one’s choices are often shaped by one’s personal biases or orientations – what one believes and values. Thus, a theory of complex behavior and decision-making must explain how and why people establish the goals they do, how knowledge is accessed in the service of those goals, and on what basis critically important (“consequential”) decisions are made. Now, here are some brief comments regarding the four categories identified above:

- **Knowledge.** One need say little about the *importance* of knowledge – which is obviously the foundation for all competent behavior! But, what *is* important is the form of knowledge organization and access, and in this arena there has been significant progress in the past decades. Psychological and educational research has shown that much routine behavior is based on the individual’s possession of knowledge “packets” that have been variously described as *scripts, schemata, or frames.* This is, in a sense, an extended form of pattern recognition: in particular contexts, we recognize certain things as being important, and “bring to mind” (through associative memory) relevant information in a form that it can be used to deal with the situation at hand. (You see a mathematics problem, recognize it as a max-min problem, and immediately know that you will obtain an analytic function, differentiate it, set the derivative equal to zero, etc.)

- **Goals.** It should be recognized that much of human behavior can be seen as *goal-oriented.* That is, we act because we want to achieve something; we choose our actions so that we can achieve the relevant goals. If we are working on solving a problem, the formal goal is to achieve a solution. Often we make a plan, which has subgoals. We work toward the subgoals, and either achieve them (in which case we move on to the next subgoal) or find alternatives. Thus, progress on a problem can be seen as the establishment of, and progress toward the achievement of, a series of goals. Complex behavior such as teaching involves trying to satisfy many goals at once.

- **Orientations.** There is a large literature on the impact of “beliefs” on problem solving performance – e.g., students who believe that mathematics word problems are merely “cover stories” for computational exercises will write down that the number of buses asked for in a problem is “31 remainder 12,” even though buses do not have remainders; or, students who believe that “mathematical proof is to confirm what you already know” will ignore proof-related results when engaged in construction problems, because they are asked to *discover* a solution, so proof is irrelevant. Thus, beliefs shape behavior. But, other things do as well – for examples one’s values. (What is more important, conceptual understanding or procedural mastery? Pure mathematics or applications? Such decisions shape curricular and instructional choices.) I have chosen “orientations” as a more neutral, more encompassing terms than beliefs, including values and personal preferences, but the category works in the same way: given a choice, one’s orientations shape one’s choices – both about which goals will be pursued, and how they will be pursued.

- **Decision-making.** There is an interesting tradition of research in economics, on *subjective* decision-making. We all know, for example, that the decision to buy a lottery ticket is a bad decision, in mathematical terms: the expected value is negative! That is, *(Probability of Winning) × (Objective value of prize) - (Cost of ticket) < 0.*
BUT, from an average person’s subjective point of view, the cost of a ticket is small (almost negligible) and the subjective value of the prize (“an easy life!”) much more than the objective value. Thus,

\[(\text{Probability of Winning}) \times (\text{Subjective value of prize}) - (\text{Subjective Cost of ticket}) > 0\]

and it is worth buying the ticket. This kind of idea can be expanded to explain a lot of human decision-making. That is, decision-making can be seen as modelable by expected-value computations, but where the quantities in the computations are the subjective values assigned by the individuals. In this way, different people will decide differently, because the subjective values they assign are different. I call this kind of decision-making the “internal calculus.” What seems crazy from the outside can be seen as (indeed, modeled as) quite rational, when viewed from the inside – IF, by rational, you mean “being a quite reasonable option, given the individual’s knowledge, goals, beliefs about and orientation to the situation.” I argue that once you understand an individual’s orientations, you can see (and model) how the individual prioritizes goals and outcomes, and see which possible courses of action have the best “yield” from his or her perspective. This allows you to model complex decision-making, such as in problem solving and teaching.

With these categories of knowledge and behavior established as background, I can now outline a theoretical view of how things fit together.

**How Things Work (in outline form)**

An individual enters into a particular context with a particular body of knowledge, goals, and orientations (beliefs, values, etc.). The individual orients to the situation. Certain pieces of information and knowledge become salient and are activated and/or triggered.

- **Goals** are established (or reinforced if they pre-existed)
- **Decisions** are made, consciously or unconsciously, in pursuit of the high-order goals.
- **Knowledge** is selected and implemented. This may be in the form of scripts, routines, schemata. Implementation begins.

Monitoring (whether effective or not) takes place on an ongoing basis.

The process iterates:

- If a subgoal is achieved, new goals kick in through the routine (or script, or…)
- If a goal is achieved, new goals kick in via decision-making.
- If the processes is interrupted or things don’t seem to be going well, decision-making kicks into action once again.

I will work through various examples of how things work, in routine and non-routine problem solving, and in teaching; I will suggest that the same theoretical architecture serves to characterize people’s behavior in activities as diverse as making breakfast (or any other meal) and in routine medical diagnosis and practice. That is, this is a theory of human behavior in broad terms.

Why do this kind of work? I will argue that theory building and testing should be a central part of the research enterprise. The more you understand something the better you can make it work. Moreover, when you understand how something skillful is done you can help others do it; hence there will be many practical applications. I will also talk about work that needs to be done – there are many ways in which the theory is still incomplete.
REFERENCES


The situational theory of problem solving attempts to explain why and how an individual communicates during a problematic situation. The situational theory of problem solving (STOPS) was proposed by Jeong-Nam Kim and James E. Grunig in 2011 through their article “problem solving and communicative action: A situational theory of problem solving.” The theory was developed from the situational theory of publics (STP) and claimed it is an extended and generalized version of STP. This theory has a goal setting theory in various ways. When goals are self-set, people with higher efficacy set higher goals than people with lower efficacy. They also are more committed to assigned goals, use better task strategies to attain goals, and respond more positively to negative feedback than people with low efficacy (Seijts & Latham, in press). These issues are addressed further below. Goal Mechanisms Goals affect performance through four mechanisms. First, goals serve a directive function; they direct attention and effort toward goal-relevant activities and away from goal-irrelevant activities. This effect occurs both cognitively and behaviorally.